



22117206



**MATHEMATICS
 HIGHER LEVEL
 PAPER 2**

Thursday 5 May 2011 (morning)

Candidate session number

2 hours

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



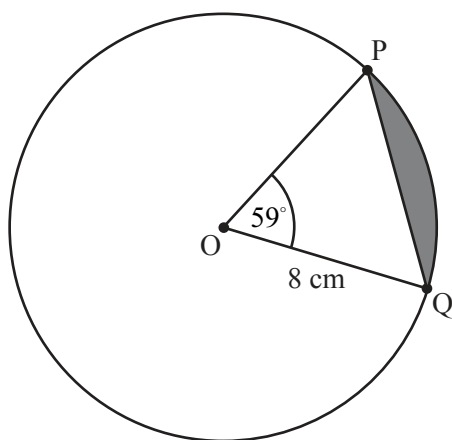
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The points P and Q lie on a circle, with centre O and radius 8 cm, such that $\hat{POQ} = 59^\circ$.



*diagram
not to scale*

Find the area of the shaded segment of the circle contained between the arc PQ and the chord [PQ].

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2. [Maximum mark: 5]

In the arithmetic series with n^{th} term u_n , it is given that $u_4 = 7$ and $u_9 = 22$.
Find the minimum value of n so that $u_1 + u_2 + u_3 + \dots + u_n > 10\,000$.

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3. [Maximum mark: 6]

A skydiver jumps from a stationary balloon at a height of 2000 m above the ground. Her velocity, $v \text{ ms}^{-1}$, t seconds after jumping, is given by $v = 50(1 - e^{-0.2t})$.

(a) Find her acceleration 10 seconds after jumping. [3 marks]

(b) How far above the ground is she 10 seconds after jumping? [3 marks]

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4. [Maximum mark: 6]

Consider the matrix $A = \begin{pmatrix} \cos 2\theta & \sin \theta \\ -\sin 2\theta & \cos \theta \end{pmatrix}$, for $0 < \theta < 2\pi$.

(a) Show that $\det A = \cos \theta$. [3 marks]

(b) Find the values of θ for which $\det A^2 = \sin \theta$. [3 marks]

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5. [Maximum mark: 7]

Sketch the graph of $f(x) = x + \frac{8x}{x^2 - 9}$. Clearly mark the coordinates of the two maximum points and the two minimum points. Clearly mark and state the equations of the vertical asymptotes and the oblique asymptote.



6. [Maximum mark: 6]

The fish in a lake have weights that are normally distributed with a mean of 1.3 kg and a standard deviation of 0.2 kg.

- (a) Determine the probability that a fish which is caught weighs less than 1.4 kg. [1 mark]
- (b) John catches 6 fish. Calculate the probability that at least 4 of the fish weigh more than 1.4 kg. [3 marks]
- (c) Determine the probability that a fish which is caught weighs less than 1 kg, given that it weighs less than 1.4 kg. [2 marks]

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7. [Maximum mark: 5]

Consider the functions $f(x) = x^3 + 1$ and $g(x) = \frac{1}{x^3 + 1}$. The graphs of $y = f(x)$ and $y = g(x)$ meet at the point $(0, 1)$ and one other point, P.

(a) Find the coordinates of P. [1 mark]

(b) Calculate the size of the acute angle between the tangents to the two graphs at the point P. [4 marks]

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8. [Maximum mark: 6]

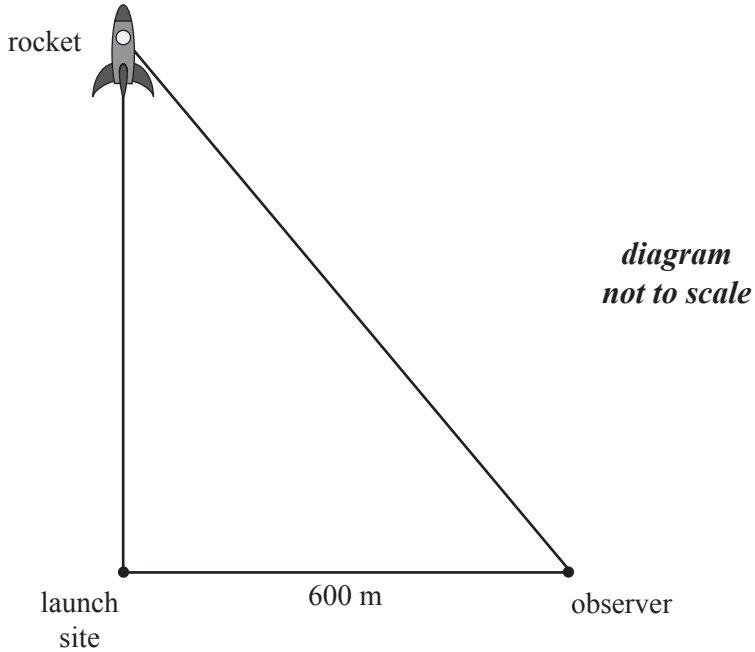
The vertices of an equilateral triangle, with perimeter P and area A , lie on a circle with radius r . Find an expression for $\frac{P}{A}$ in the form $\frac{k}{r}$, where $k \in \mathbb{Z}^+$.

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9. [Maximum mark: 6]

A rocket is rising vertically at a speed of 300 ms^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.



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10. [Maximum mark: 8]

The point P, with coordinates (p, q) , lies on the graph of $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$, $a > 0$.
The tangent to the curve at P cuts the axes at $(0, m)$ and $(n, 0)$. Show that $m + n = a$.

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Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

SECTION B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

11. [Maximum mark: 17]

The points $P(-1, 2, -3)$, $Q(-2, 1, 0)$, $R(0, 5, 1)$ and S form a parallelogram, where S is diagonally opposite Q .

- (a) Find the coordinates of S . [2 marks]
- (b) The vector product $\vec{PQ} \times \vec{PS} = \begin{pmatrix} -13 \\ 7 \\ m \end{pmatrix}$. Find the value of m . [2 marks]
- (c) Hence calculate the area of parallelogram $PQRS$. [2 marks]
- (d) Find the Cartesian equation of the plane, Π_1 , containing the parallelogram $PQRS$. [3 marks]
- (e) Write down the vector equation of the line through the origin $(0, 0, 0)$ that is perpendicular to the plane Π_1 . [1 mark]
- (f) Hence find the point on the plane that is closest to the origin. [3 marks]
- (g) A second plane, Π_2 , has equation $x - 2y + z = 3$. Calculate the angle between the two planes. [4 marks]



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

12. [Maximum mark: 18]

The number of accidents that occur at a large factory can be modelled by a Poisson distribution with a mean of 0.5 accidents per month.

- (a) Find the probability that no accidents occur in a given month. [1 mark]

- (b) Find the probability that no accidents occur in a given 6 month period. [2 marks]

- (c) Find the length of time, in complete months, for which the probability that at least 1 accident occurs is greater than 0.99. [6 marks]

- (d) To encourage safety the factory pays a bonus of \$1000 into a fund for workers if no accidents occur in any given month, a bonus of \$500 if 1 or 2 accidents occur and no bonus if more than 2 accidents occur in the month.
 - (i) Calculate the expected amount that the company will pay in bonuses each month.

 - (ii) Find the probability that in a given 3 month period the company pays a total of exactly \$2000 in bonuses. [9 marks]



Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.

13. [Total mark: 25]

Part A [Maximum mark: 8]

Prove by mathematical induction that, for $n \in \mathbb{Z}^+$,

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \dots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}.$$

Part B [Maximum mark: 17]

(a) Using integration by parts, show that $\int e^{2x} \sin x \, dx = \frac{1}{5} e^{2x} (2 \sin x - \cos x) + C$. [6 marks]

(b) Solve the differential equation $\frac{dy}{dx} = \sqrt{1-y^2} e^{2x} \sin x$, given that $y = 0$ when $x = 0$, writing your answer in the form $y = f(x)$. [5 marks]

(c) (i) Sketch the graph of $y = f(x)$, found in part (b), for $0 \leq x \leq 1.5$. Determine the coordinates of the point P, the first positive intercept on the x -axis, and mark it on your sketch.

(ii) The region bounded by the graph of $y = f(x)$ and the x -axis, between the origin and P, is rotated 360° about the x -axis to form a solid of revolution. Calculate the volume of this solid. [6 marks]

